Heitziau Dipole

$\vec{A_s}$ in far feild

$$\vec{\nabla} \times \vec{A} = \vec{H}$$

$$\vec{\nabla} \times \vec{H} = \vec{E}$$

$$e^{-jkz_s\cos\theta}$$

dismissed retardation

$$H_{s\phi} = \frac{E_{s\theta}}{\eta}$$

pointing vector

$$\langle \vec{P}(R_p) \rangle = \frac{1}{2} \mathrm{Re}\left[\vec{E_s} \times \vec{H_s}^* \right]$$

$$= \frac{1}{2} \mathrm{Re}\left[E_{s\theta}\, \hat{\theta} \times \hat{\phi}\, H_{s\phi}^* \right]$$

$$= \frac{1}{2} \frac{|E_{s\theta}|^2}{\eta_0} \hat{r}$$

$$= \hat{r} \left[\frac{I_0 (dl)^2}{32\pi R_p^2} \right] k^2 \eta_0 \sin^2\theta$$

Greatest at $\theta = \pi/2$

$$\langle \vec{P} \rangle = \langle P_R \rangle \hat{r}$$

$$\langle P_r \rangle_{max} = \frac{15\pi I_o^2}{R_p^2}\left(\frac{d\ell}{\lambda}\right)^2$$

$$\text{for} \quad \eta_o = 120\pi$$
$$k = \frac{2\pi}{\lambda}$$

$$U(\theta, \varphi) = R_p^2 \langle P_R \rangle$$

Net radiated power

$$P_{NET} = \int R_p^2 \sin\theta \, d\theta \, d\phi \, \langle P_R \rangle$$

surface element

$$= 15\pi^2 I_o^2 \left(\frac{d\ell}{\lambda}\right)^2 \int_0^{2\pi} d\phi \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta$$

$$= 40\pi^2 I_o^2 \left(\frac{d\ell}{\lambda}\right)^2$$

$\left(\frac{d\ell}{\lambda}\right) <<< 1 \quad \therefore$ it is convienent

$$P_{NET} = \left[R_{rad}\right] I_o^2$$

$$R_{rad} = \frac{P_{NET}}{I_0^2} = 40\pi\left(\frac{dl}{\lambda}\right)^2$$

$$P_{NET} = \int R_P^2 \sin\theta \, d\theta \, d\phi \, \langle P_R \rangle$$

$$= \int \sin\theta \, d\theta \, d\phi \, u(\theta, \varphi)$$

$$\langle u \rangle = \frac{1}{4\pi} \int \sin\theta \, d\theta \, d\varphi \, u(\theta, \varphi)$$

$$G_d = \frac{u(\theta, \varphi)}{\langle u \rangle}$$

$$D = \frac{u(\theta, \varphi)}{\langle u \rangle}_{max}$$

$D = 1$ for isotropic radiation.

$D_{Hertzian} = 3/2$

Finite Antennas (far feild)

$$d\vec{E}_s = \hat{\theta}\, j \frac{k\eta_0}{4\pi} \frac{I(z_s)\,dz_s}{|\vec{R}_p - \vec{R}_s|} e^{-j|R_p - R_s|} \sin\theta_p$$

for the hertzian dipole: $I(z_s) = I_0$

constant.

$$|\vec{R}_p - \vec{R}_s| \sim R_p - R_s \cos\theta_p + \dots$$

$$= R_p - z_p \cos\theta_p$$

half wave dipole:

$$L = \frac{d}{2}$$

$$I(z_0, t) = I_0 \cos\omega t \cos kz_s$$

$$I_s = I_0 \cos kz_s$$

$$\frac{1}{|\vec{R}_p - \vec{R}_s|} = \frac{1}{R_p\left(1 - \frac{z_s}{R_p}\cos\theta_p\right)} \sim \frac{1}{R_p}$$

$$e^{-jk|\vec{R}_p - \vec{R}_s|} = \cos\left(k\left(R_p - z_s\cos\theta\right)\right)$$

$R_p >>> z_s$

$\therefore \quad \cos(k z_s \cos\theta)$

$$e^{-jk|\vec{R}_p - \vec{R}_s|} = e^{-jkR_p} e^{-jkz_s\cos\theta}$$

$$E_{\theta s} = \int_{-d/4}^{d/4} dE_{\theta s}$$

$$= \int_{-d/4}^{d/4} dz_s \left[j\frac{\eta_0 k I_0 \sin\theta_p}{4\pi R_p} e^{-jkR_p} \right.$$

$$\left. * \cos kz_s \, e^{-jkz_s\cos\theta} \right]$$

$$= j\frac{k\eta_0 I_0 \sin\theta_p}{4\pi R_p} e^{-jkR_p} \int_{-d/4}^{d/4} dz_s \cos kz_s \, e^{-jkz_s\cos\theta}$$

$$\int_{-d/4}^{d/4} dz_s \cos kz_s \, e^{-jkz_s\cos\theta_p}$$

$$= \frac{2}{k\sin^2\theta_p} \cos\left[\frac{\pi}{2}\cos\theta_p\right]$$

$$\boxed{\frac{kd}{4} = \frac{\pi}{2}}$$

$$E_{s\theta} = j\frac{60 I_o}{R_p \sin\theta_p}\cos\left(\frac{\pi}{2}\cos\theta_p\right)$$

$$H_{s\varphi} = \frac{E_{s\theta}}{\eta_o} \qquad \eta_o = 120\pi$$

$$\langle \vec{P} \rangle = \frac{1}{2}\hat{r}\,\mathrm{Re}\left[E_{s\theta} H_{s\varphi}^{*}\right]$$

$$= \hat{r}\,\frac{15 I_o^2}{\pi R_p^2}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta_p\right)}{\sin\theta_p}\right]^2$$

$$U(\theta,\varphi) = \frac{15 I_o^2}{\pi}\left(\frac{\cos\left(\frac{\pi}{2}\cos\theta_p\right)}{\sin\theta_p}\right)^2$$